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# Real-Time Reactive Power Distribution in Microgrids by Dynamic Programming

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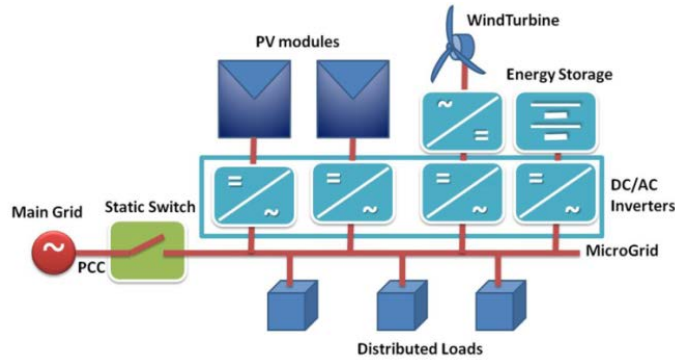
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**Abstract:** In this paper a new real-time optimization method for reactive power distribution in microgrids is proposed. The method enables location of a globally optimal distribution of reactive power under normal operating conditions. The method exploits the typical compact structure of microgrids to obtain a solution by parts, using the dynamic programming method and Bellman equation. The proposed solution method is based on the fact that the microgrid is designed with a central feeder line to which clusters of generators and loads are connected, and is suitable for microgrids with ring topologies as well as radial ones. The optimization problem is formulated with the cluster reactive powers as free variables, and the solution space is spanned by the cluster reactive power outputs. The optimal solution is then constructed by efficiently scanning the entire solution space, by scanning every possible combination of reactive powers, by means of dynamic programming. Since every single step involves a one-dimensional problem, the complexity of the solution is only linear with the number of clusters, and as a result, a globally optimal solution may be obtained in real time. The paper includes the results of two test-case networks.

## 1. Introduction

Microgrids are small power networks that integrate local loads and local power sources [1]-[3]. Fig.1 illustrates a typical microgrid network that includes renewable generators and storage devices, and connects to the utility grid at the Point of Common Coupling (PCC). The locality of the microgrid network enables improved energy management, and hence improved stability and efficiency [4]-[7]. The objectives of the microgrid management system depend on the microgrid mode of operation: islanded or grid-connected. In islanded mode, possible objectives are to stabilize the system's frequency and voltage [8]-[9] or when working in autonomous mode, to minimize the consumption of fuel . In grid-connected mode, typical objectives are to minimize the fuel, total emissions, total power loss, or as considered in this work, to optimize the reactive power flow at the microgrid PCC [10]-[12].



**Fig.1.** A typical microgrid, integrating a variety of energy sources.

One open challenge in microgrid management is the internal distribution of reactive power [13]. A common approach is to solve reactive power distribution problems by means of gradient based solvers, such as Newton's method [14] and Successive Quadratic Programming (SQP) [15], which essentially uses the gradient of the objective function to locate a local minimum. Another approach is the interior point method [16] which may handle inequalities well, and is typically efficient in large-scale problems. Linear programming may also be used, if the problem is known to be linear in the vicinity of its operating point [17]. An additional emerging approach is to employ probabilistic search methods, such as particle swarm optimization and genetic algorithms [18], [19].

A primary disadvantage of the methods reviewed above is that they are not guaranteed to converge to the objective function global optimum. In optimal reactive power flow problems, this means that the best dispatch of generators and storage devices is often unknown, especially when the numeric solver should converge in real-time.

In light of this gap, this paper proposes a new optimization method named MRODP (Microgrid Reactive Power Optimization by Dynamic Programing) that is specifically tailored for microgrids, and can locate a globally optimal power flow solution under normal operating conditions. The proposed method is based on a dynamic programming algorithm (reviewed in section 2), and exploits the typical compact structure of microgrids to obtain a solution by parts, using the dynamic programming Bellman equation. The proposed solution method is based on the following principle: The microgrid is designed with a central feeder line to which clusters of generators and loads are connected. This structure is suitable for ring topologies as well as radial ones, and is therefore applicable to most microgrids. Nevertheless, there might be other topologies for which this arrangement is not be valid. Two quantities are assigned with each cluster - its total active power, and total reactive power. Using this structure, the optimization

problem is formulated with the cluster reactive powers as free variables, and the solution space is spanned by the cluster reactive powers. The optimal solution is then constructed by efficiently scanning the entire solution space, by scanning every possible combinations of reactive powers, until locating the optimal power flow and the reactive power import. However, instead of directly scanning every possible combination, a process which numerical complexity may be impractical, the solution space is scanned in consecutive steps, using a dynamic programming algorithm and the Bellman equation. At every step, the algorithm scans the reactive power of a single cluster, computing the cluster internal power flow for every value of the cluster reactive power. At the end of this process, the reactive power import at the PCC is known for every value of reactive power in each of the clusters. These results are then merged by the Bellman equation, to locate the microgrid optimal power flow.

The major advantage of the processes described above is that the optimal power flow is calculated only once at every cluster. Since every single step involves a one-dimensional problem, the complexity of the solution is only linear with the number of clusters, and as a result, a globally optimal solution may be obtained in real time.

This paper is organized as follows. A short review about dynamic programming, which is the basis of the approach presented in this paper, is presented in section 2. Microgrids modelling along with the formulation of the optimization problem are detailed in section 3. Section 4 presents in detail the 5-step optimization procedure. In sections 6 and 7 two different test cases are optimized by the proposed procedure. In section 6 a ring type microgrid is considered while in section 7 a radial one is investigated. Section 8 concludes the paper.

## 2. DYNAMIC PROGRAMING - A SHORT REVIEW

Dynamic Programming [20],[21] is an optimization method that solves complex problems by dividing them into simpler sub-problems. This method may be used only if an objective function can be expressed by a recursive formula, which separates it to simpler functions of lower dimension. Thus, to solve a complex problem using the dynamic programming approach, one needs to solve different parts of the problem, and then merge the partial solutions. A trivial example is given by the following optimization problem:

$$\max_{x_1 \dots x_N} \left\{ \sum_{i=1}^N g_i(x_i) \right\} \quad (1)$$

Where  $g_i(x_i)$  are arbitrary, non-convex, one-dimensional functions. With no constraints, different  $x_i$  variables do not affect each other, and so this problem can be divided into smaller sub-problems, in which every  $x_i$  is solved separately:

$$\max_{x_1 \dots x_N} \left\{ \sum_{i=1}^N g_i(x_i) \right\} = \sum_{i=1}^N \max_{x_i} \{ g_i(x_i) \} \quad (2)$$

In this example, the high-dimensional problem is transformed into a series of simpler problems, and the complexity of the solution is substantially reduced. In addition, if the global optimum of every function  $g_i(x_i)$  can be found, the overall optimal solution, which is simply the collection of the partial solutions, is also known.

Dynamic programming problems are often more complex than this simple example, because the sub-problems may involve more than one variable, or the problem may include constraints than involve several variables. Such problems are efficiently solved if they can be expressed by a recursive formula that divides them into smaller sub-problems. As an example, in the following problem the sum of the variables is a constraint:

$$\max_{x_1 \dots x_N} \left\{ \sum_{i=1}^N g_i(x_i) \right\} \quad \text{subjected to } 0 \leq x_i \leq X_{\max} \quad \text{and} \quad \sum_{i=1}^N x_i \leq S_{\max} \quad (3)$$

where  $X_{\max}$  and  $S_{\max}$  are constants. If the functions  $g_i$  are not convex, this problem may be hard or impossible to solve using gradient based methods. However, the problem is efficiently solved by a dynamic programming algorithm, by dividing it into smaller problems using recursion. To formulate a recursive expression, a cost function  $V$  is defined:

$$V_k(s_k) = \max_{x_i} \left\{ \sum_{i=1}^k g_i(x_i) \right\} \quad \text{subjected to } 0 \leq x_i \leq X_{\max} \quad \text{and} \quad \sum_{i=1}^k x_i = s_k \quad (4)$$

And the original problem is expressed as:

$$V_k(s_k) = \max_{x_i} \{ V_{k-1}(s_k - x_i) + g_k(x_i) \} \quad \text{subjected to } 0 \leq x_i \leq X_{\max} \quad \text{and} \quad x_i \leq s_k \quad (5)$$

This type of recursion is often called the Bellman equation. The original problem is expressed as a series of steps, where each step involves only a single variable  $x_i$ . Using the Bellman equation, the original problem may be solved in parts, where each part involves optimization over the single variable  $x_i$ . The solution process starts at  $k=1$ , for which the cost function is known  $V_1(s_1) = g_1(s_1)$ . Then, for every  $k=2 \dots N$ , the cost  $V_k(s_k)$  may be computed for every value of  $s_k$ . At the end of this process, the optimum of the original problem is obtained by the maximum of  $V_N(S_N)$ :

$$\max_{x_1 \dots x_N} \left\{ \sum_{i=1}^N g_i(x_i) \right\} = \max_{0 \leq s_N \leq s_{\max}} \{V_N(s_N)\} \quad (6)$$

This solution is numerically efficient, as its complexity increases linearly with the number of variables. In addition, since this algorithm scans over the entire solution space, it is guaranteed to locate the global optimum.

### 3. Microgrid modelling

The microgrid considered in this paper consists of  $N$  buses, which are indexed  $i=1..N$ . Several variables are associated with each bus  $i$ :

- $P_{g,i}/Q_{g,i}$ – Active and reactive power generation.
- $P_{L,i}/Q_{L,i}$ – Active and reactive power consumption (load).
- $V_i$  – the voltage magnitude of the bus.
- $\delta_i$  – the voltage phase angle.

Lines are notated as follows:  $Y_{i,k}$  is the admittance magnitude, associated with the line connecting bus  $i$  to bus  $k$ . The admittance matrix  $Y$  includes all additional components such as line to ground capacitance, voltage regulation capacitors, transformers tap etc.. The admittance phase is  $\psi_{i,k}$ . Line currents are notated  $I_{i,k}$ . Power flow is governed by the power flow equations (See eq.(7)). The microgrid is assumed grid-connected, and is linked to the public grid through the Point of Common Coupling (PCC), which is considered the ‘slack-bus’ of the system. The phase angle of the PCC is defined zero ( $\delta_{PCC}=0$ ), and serves as a reference. PCC powers are notated  $P_{PCC}$  and  $Q_{PCC}$ . Power factor at the PCC is notated  $\theta_{PCC}$ .

The optimizer obeys the following guidelines:

- Desired reactive power at the PCC. The power factor,  $\cos(\theta_{PCC})$  is controlled as desired.
- Power flow constraints. Voltages, currents, and powers must all strictly maintain specified limits given in (7).
- Real time computation. The reactive power distribution must be recomputed for each combination of power generation and load. The solver must converge fast, within milliseconds to seconds.

The objectives and constraints are summarized as follows:

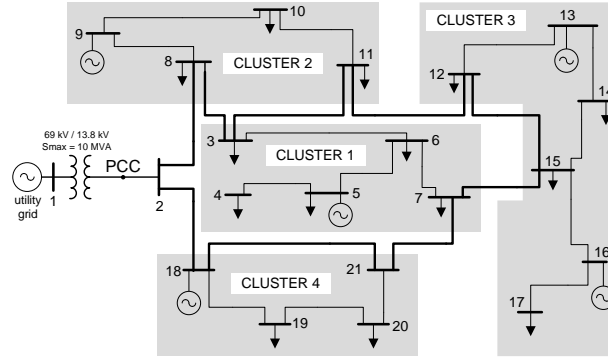
$$\begin{aligned}
&\text{Objective: } \min \left\{ \left( \cos(\Theta_{PCC}) - \text{ref} \right)^2 \right\} \\
&\text{Constraints: (power flow equations)} \\
&P_{g,i} - P_{L,i} = V_i \cdot \sum_{k=1}^N Y_{i,k} \cdot V_k \cdot \cos(\delta_i - \delta_k - \psi_{i,k}) \\
&Q_{g,i} - Q_{L,i} = V_i \cdot \sum_{k=1}^N Y_{i,k} \cdot V_k \cdot \sin(\delta_i - \delta_k - \psi_{i,k}) \\
&I_{i,k} = Y_{i,k} \cdot \left| V_i e^{j\delta_i} - V_k e^{j\delta_k} \right| \\
&\text{Reactive power at PCC: } Q_{PCC}^{\min} \leq Q_{PCC} \leq Q_{PCC}^{\max} \\
&\text{voltage limits: } V_i^{\min} \leq V_i \leq V_i^{\max} \quad (\forall i) \\
&\text{current limits: } I_{i,k}^{\min} \leq I_{i,k} \leq I_{i,k}^{\max} \quad (\forall i, k) \\
&\text{Reactive powers: } Q_i^{\min} \leq Q_{g,i} \leq Q_i^{\max} \quad (\forall i) \\
&\text{Apparent powers: } \left( S_i^{\min} \right)^2 \leq P_{g,i}^2 + Q_{g,i}^2 \leq \left( S_i^{\max} \right)^2
\end{aligned} \tag{7}$$

In these equations,  $\text{ref}$  is the desired power factor at PCC, the free variables are  $Q_{g,i}$ , the reactive powers of generators. All other variables are uncontrollable:  $P_{g,i}, P_{L,i}, Q_{L,i}$  are measured inputs.  $P_{PCC}, Q_{PCC}, V_i, \delta_i, I_{i,k}$  emerge from the power-flow equations.

#### 4. The Optimization Process of MRODP

This work addresses the following optimization problem: find the reactive powers of generators that optimize the power factor at the PCC, given the voltage, current and power limits of the network. A first step is to divide the microgrid into smaller clusters. Recent microgrid studies ([1], [22], [23], [24] and a review in [4]), indicate that microgrids are not arbitrarily meshed topologies, but in fact designed with a typical structure: peripheral clusters that are assembled around a central PCC. Examine, for instance, the study system in [22], where this clustered structure is explicitly shown. In this microgrid, the network is divided into clusters connected by feeder lines to a central PCC. The clustered structure simplifies the problem, as it enables to solve it using a dynamic programming algorithm. It should be mentioned here that various possibilities for clustering the microgrid may exist. Moreover, even if a microgrid can be clustered in several ways, the dynamic programming procedure ensures the same optimal solution, since it scans the entire solution space. An open question that is not discussed here is the question of optimal clustering, which may lead to a reduction of computational complexity.

Fig. 3a shows the clusters with their respective power signals. The number of clusters is  $M$ . Powers supplied to the cluster  $j$  are notated  $P_{clusj}, Q_{clusj}$ . Within each cluster, the powers associated with the cluster  $P_{clusj}, Q_{clusj}$  may be evaluated by solving the local power flow equations within the cluster. Solutions are obtained while shorting all external buses to the PCC, which is modeled as a voltage source with a zero phase.



**Fig.1.** A Generic clustered Microgrid case study

Doing so, the cluster powers are obtained as functions of the reactive controller:  $P_{clusj}(Qg_{clus,j})$ ,  $Q_{clusj}(Qg_{clus,j})$ . This process is illustrated in Fig. 4, which shows the equivalent topology for solving cluster 4 of Fig.2. Using clusters terminology, the initial optimization problem of (7) may now be concisely formulated:

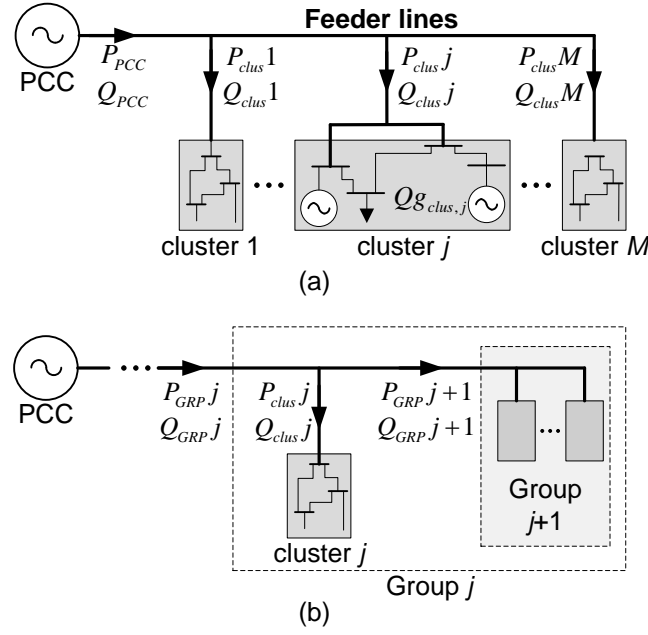
$$\begin{aligned} \text{objective: } & \cos^2(\Theta_{PCC}) \rightarrow \max \\ \text{constraint: } & Q_{PCC}^{\min} \leq \sum_{j=1}^M Q_{clus,j}(Qg_{clus,j}) \leq Q_{PCC}^{\max} \end{aligned} \quad (8)$$

The method presented here is not confined to unity power factor only. The objective function in (2) can easily include a desired limit instead of maximizing the power factor, and then the solution will be confined to these requirements. This might be useful when considering the microgrid as a source of reactive power compensation to the main grid. Additional constraints are the power flow limits (voltages, currents, powers) within each cluster. These are easily handled by assigning infinity active power values to infeasible cluster solutions, as follows:

$$P_{clus,j}(Qg_{clus,j}) = \begin{cases} \text{feasible} & (\text{power-flow solution}) \\ \text{infeasible} & \infty \end{cases} \quad (9)$$

To formulate a recursive dynamic programming solution, groups of clusters are defined: A group  $j$  is defined as the set of clusters  $j...M$ . Alternatively, a group may be defined recursively: group  $j$  contains cluster  $j$ , and all the clusters in group  $j+1$  (see Fig. 3b). This recursive definition is a key to the solution: The optimal reactive power entering group  $j$  is a combination of reactive power at cluster  $j$  and group  $j+1$ . Thus, the procedure in (9) can be summarized as follows: for each cluster the power flow solution provides specific values for active and reactive powers, voltages and currents. If these values are within the limits as specified in (7) then the solution is feasible and a cost is calculated accordingly. If not the cost is assigned a value of infinity.





**Fig. 2.** The solution process. (a) Approximated network topology, assuming that the clusters are shorted to the PCC. (b) Recursive cluster grouping.

The explicit formulation of the combination of reactive power at cluster  $j$  and group  $j+1$  is given in (10) :

$$\begin{aligned} P_{GRP,j} &= \sum_{k=j}^M P_{clus,k} (Qg_{clus,k}) \\ Q_{GRP,j} &= \sum_{k=j}^M Q_{clus,k} (Qg_{clus,k}) \end{aligned} \quad (10)$$

For every value of  $Q_{GRP,j}$ , the optimal active power is defined as follows:

$$\begin{aligned} P_{GRP,j}^{opt}(Q_{GRP,j}) &= \min_{Qg_{clus,j}, \dots, Qg_{clus,M}} \{P_{GRP,j}\} \\ \text{subjected to: } &\sum_{k=j}^M Q_{clus,k} (Qg_{clus,k}) = Q_{GRP,j} \\ Qg_{clus,j}^{opt} \dots Qg_{clus,M}^{opt} &= \arg \min \{ \text{same expression} \} \end{aligned} \quad (11)$$

The function  $P_{GRP,j}^{opt}(Q_{GRP,j})$  is named the active-reactive function. It measures the minimal active power that must be supplied to group  $j$  in order to maintain a reactive power flow of  $Q_{GRP,j}$ .  $Qg_{clus,j}^{opt} \dots Qg_{clus,M}^{opt}$  are the corresponding optimal reactive controllers of generators within the group. Employing these definitions, the solution to the optimization problems in (7) and (8) is equal to the optimal power of group 1:

$$\begin{aligned} \cos^2(\Theta_{PCC}) &= \frac{(P_{PCC})^2}{(P_{PCC})^2 + (Q_{PCC})^2} \rightarrow \max \\ P_{PCC} &= \min_{Q_{PCC}^{\min} \leq Q_{GRP,1} \leq Q_{PCC}^{\max}} \{P_{GRP,1}^{opt}(Q_{GRP,1})\} \\ Q_{PCC} &= \arg \min_{Q_{PCC}^{\min} \leq Q_{GRP,1} \leq Q_{PCC}^{\max}} \{P_{GRP,1}^{opt}(Q_{GRP,1})\} \end{aligned} \quad (12)$$

The challenge is therefore to compute the active-reactive functions  $P_{GRP}^{opt}(Q_{GRPj})$  for each group  $1 \dots M$ . This is done using a fast recursive dynamic programming algorithm involving five stages, which are described next.

### Stage 1: Initiation

The active-reactive function of the last group  $M$  is computed first. This group contains only one cluster (cluster  $M$ ), thus group powers are equal to cluster powers:

$$\begin{aligned} \forall Q_{g_{clus,M}} : P_{GRPM}^{opt} &= P_{clus,M}(Q_{g_{clus,M}}) \\ Q_{GRPM} &= Q_{clus,M}(Q_{g_{clus,M}}) \end{aligned} \quad (13)$$

### Stage 2: Backward recursion

Consider the recursive formulation, described in Fig. 3b, and eq.(11). Given that the active-reactive function is known for group  $j+1$ , it may be directly computed for group  $j$ :

and therefore the various powers can be written as:

$$P_{GRPj}^{opt}(Q_{GRPj}) = \min_{Q_{g_{clus,j}}} \left\{ P_{clus,j}(Q_{g_{clus,j}}) + P_{GRP,j+1}^{opt}(Q_{GRPj} - Q_{clus,j}(Q_{g_{clus,j}})) \right\} \quad (14)$$

For example, using the known function  $P_{GRP}^{opt}M(Q_{GRPM})$  at group  $M$  enables a computation of the function at group  $M-1$ . Notice that minimization is carried out over a single variable:  $Q_{g_{clus,j}}$ . The process continues from group  $M$  down to group 1 (the PCC). This is illustrated at Fig. 6a. At the end of this stage, all active-reactive functions are known.

### Stage 3: Choosing a final point

At this stage, the optimal combination of active and reactive power is selected at the PCC. Powers at the PCC are identical to powers of group 1, which has been computed during the backward recursion stage. From the available and finite values of  $P_{GRP}^{opt}1(Q_{GRP1})$ , one optimal point should be selected. (If all values are infinite, the network has no valid solution within constraints). We choose the point that maximizes the power factor:

$$\begin{aligned} \cos^2(\Theta_{PCC}) &= \frac{(P_{GRP1}^{opt}(Q_{GRP1}))^2}{(P_{GRP1}^{opt}(Q_{GRP1}))^2 + (Q_{GRP1})^2} \rightarrow \max \\ \text{s.t. : } Q_{PCC}^{\min} &\leq Q_{GRP1} \leq Q_{PCC}^{\max} ; P_{GRP1}^{opt}(Q_{GRP1}) \text{ is finite} \end{aligned} \quad (15)$$

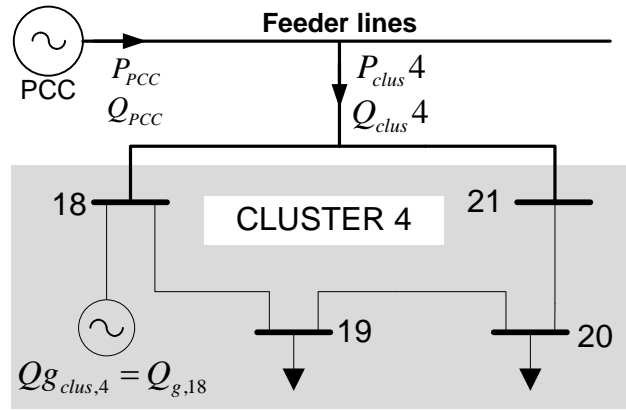
The selected value of reactive power at the PCC is notated  $Q_{GRP}^{opt}1$ . This is the optimal reactive power at group 1.

#### Stage 4: Forward recursion

The forward recursion starts at group 1 (the PCC), and continues up to group  $M$ . At each phase, known values of the optimal reactive power at group  $j$ ,  $Q_{GRPj}^{opt}$ , and the known active-reactive function at group  $j+1$ ,  $P_{GRP,j+1}(Q_{GRP,j+1})$  are used for computing optimal values at cluster  $j$ :

$$\begin{aligned} Q_{g,clus,j}^{opt} &= \arg \min_{Q_{g,clus,j}} \left\{ P_{clus,j}(Q_{g,clus,j}) \right. \\ &\quad \left. + P_{GRP,j+1}^{opt}(Q_{GRPj}^{opt} - Q_{clus,j}(Q_{g,clus,j})) \right\} \\ Q_{GRP,j+1}^{opt} &= Q_{GRPj}^{opt} - Q_{clus,j}(Q_{g,clus,j}^{opt}) \end{aligned} \quad (16)$$

$Q_{g,clus,j}^{opt}$  is the optimal reactive controller at cluster  $j$ .  $Q_{GRP,j+1}^{opt}$  is the optimal reactive power at group  $j+1$ . At the end of this stage, all optimal reactive controllers are known. Forward recursion is illustrated at Fig. 5b.



**Fig. 3.** Local power flow solution of cluster 4. Power inputs ( $P_{clus,4}$ ,  $Q_{clus,4}$ ) are evaluated as a function of ( $Q_{g,18} = Q_{g,clus,4}$ ). Power flow is solved within the cluster.

#### Stage 5: validation of the approximated solution

As the reactive power of each generator is explicitly found using the forward recursion stage while it computes all reactive controllers, it is possible to validate the initial cluster and short circuit assumptions. Using the explicit reactive power of each generator, the full power flow of the entire network may be directly computed. This explicit solution validates the approximated solution that has been acquired by clustering, since it includes the non-zero impedances of the feeder lines.

It should be noted, that if the impedance of the feeder line in a specific microgrid should not have been neglected, this stage will indicate that the short circuit approximation should not be used in the given microgrid.

The optimization process as described in this section is summarized in the flow chart as shown in Fig. 4.

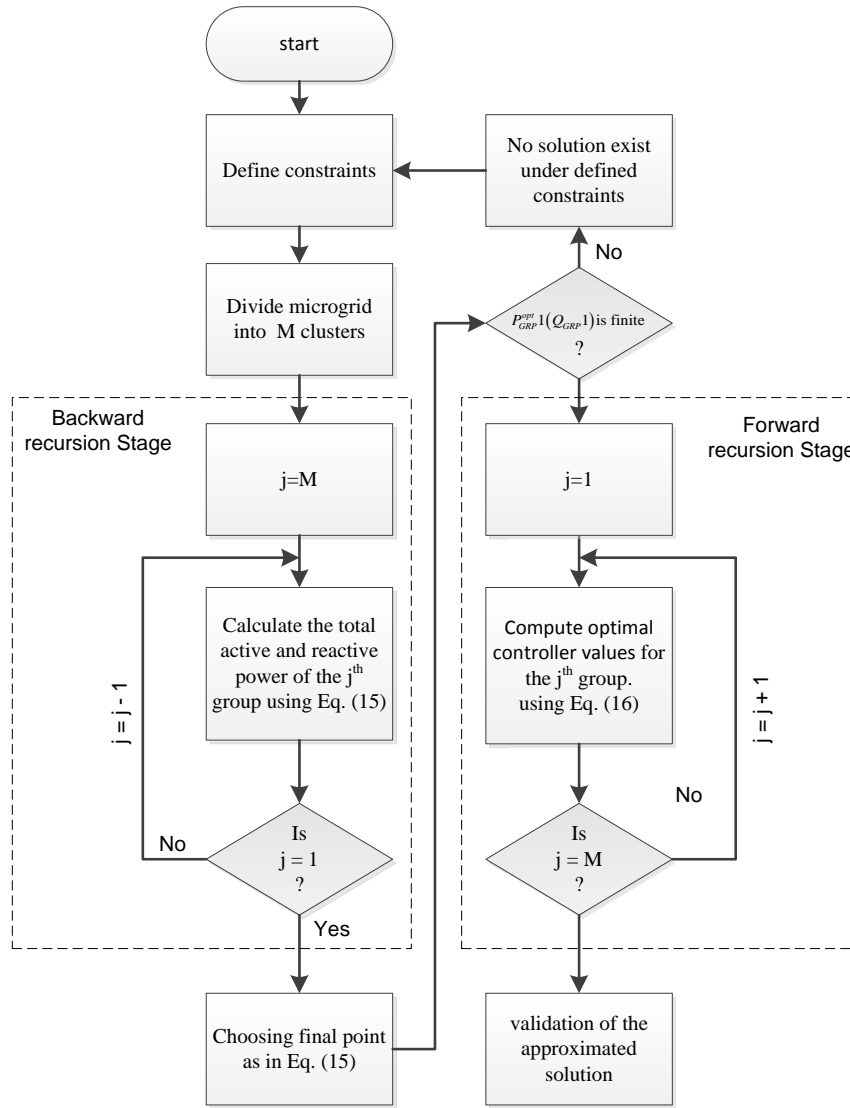


Fig. 4. The optimization process flow chart

## 5. Microgrid case-study

The optimal reactive power distribution using MRODP is evaluated in a microgrid case-study network in Fig. 7. The topology is a “ring”, fed by a transformer rated at 13.8 kV and 10 MVA at its secondary side. The transformer secondary is considered the network’s PCC. Full network data is presented at Table 1. Load powers and active generated powers are randomized over a 72 hr period. All values are given over a per-unit (p. u.) base. The specific powers shown at Table 1 are sampled at time  $t = 36$  hr. For each time point, reactive power at the PCC is optimized for power factor (eq. **Error! Reference source not found.**), over a regulatory range of  $-0.4 \leq Q_{PCC} \leq 0.4$  (in p.u.).

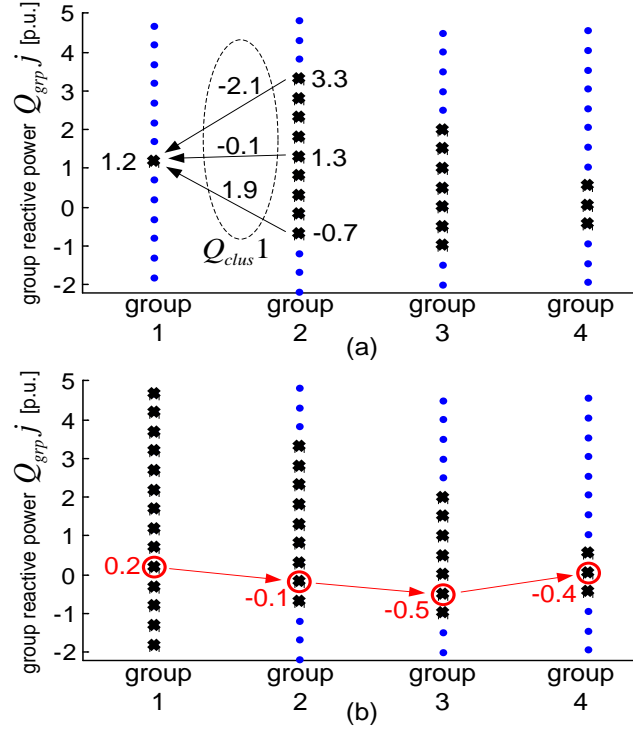
The network is modeled using the clustered approximation. The feeder lines ( $2 \rightarrow 8$ ,  $8 \rightarrow 3$ ,  $3 \rightarrow 11 \dots$ ) are considered an electric short, and the four clusters ( $M=4$ ) are considered shorted to the PCC. At each time point, the reactive power output of the five network's generators ( $Q_{g5}$ ,  $Q_{g9}$ ,  $Q_{g13}$ ,  $Q_{g16}$ ,  $Q_{g18}$ ) is computed using the five-stage dynamic programming solution. Local power flow solutions in clusters are computed using the simple Gauss-Seidel method, using a free software tool. Computation is done over a numerical time-power grid, using time steps of  $\Delta t=0.1$  hr and  $\Delta Q=0.1$  [p.u.]. Equations (14)-(16) require computation of values that reside between points of the numerical grid. These values are computed by interpolation of grid points (see Fig. 5).

Table 1 - MICROGRID DATA

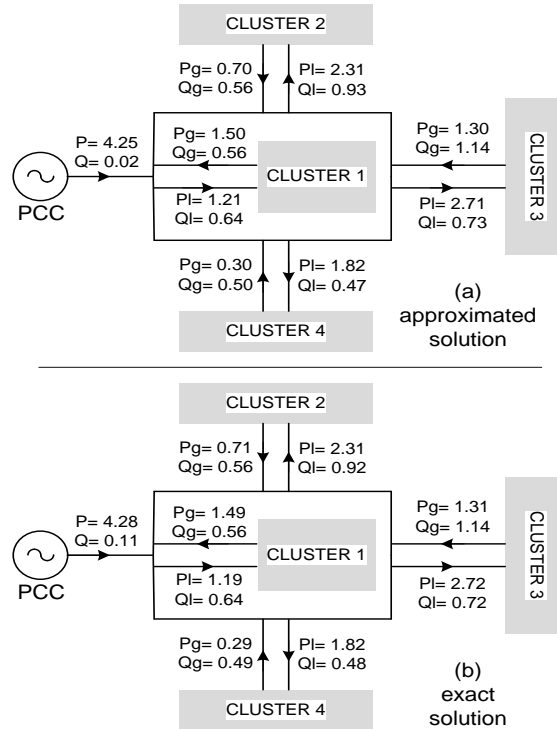
per-unit base (p.u.)							
S <sub>base</sub> = 1 MVA   V <sub>base</sub> = 13.8 kV   I <sub>base</sub> = 72.46 A   Z <sub>base</sub> = 190.44 Ω							
LOADS							
bus	active power P <sub>L</sub>	Reactive power Q <sub>L</sub>		Vol. Min	Vol. max		
2	0	0		0.95	1.05		
3	0.2	0.0		0.95	1.05		
4	0.3	0.1		0.95	1.05		
5	0.2	0.1		0.95	1.05		
6	0.4	0.2		0.95	1.05		
7	0.1	0.2		0.95	1.05		
8	0.7	0.3		0.95	1.05		
9	0	0		0.95	1.05		
10	1.0	0.4		0.95	1.05		
11	0.6	0.2		0.95	1.05		
12	0.5	0.1		0.95	1.05		
13	0	0		0.95	1.05		
14	0.3	0.0		0.95	1.05		
15	1.2	0.2		0.95	1.05		
16	0	0		0.95	1.05		
17	0.7	0.4		0.95	1.05		
18	0	0		0.95	1.05		
19	1.0	0.2		0.95	1.05		
20	0.8	0.2		0.95	1.05		
21	0	0					
GENERATORS							
bus	active power P <sub>g</sub>	reactive power range Q <sub>min</sub> ... Q <sub>max</sub>		apparent power limit S <sub>max</sub>	Vol. min. V <sub>min</sub>	Vol. Max V <sub>max</sub>	
2 (PCC)	1	-0.4 ... 0.4		10	0.95	1.05	
5	1.5	-1.5 ... 1.5		2.5	0.95	1.05	
9	0.7	-0.6 ... 0.6		1.0	0.95	1.05	
13	0.7	-0.6 ... 0.6		1.2	0.95	1.05	
16	0.6	-0.6 ... 0.6		1.2	0.95	1.05	
18	0.3	-0.5 ... 0.5		1.0	0.95	1.05	
LINES							
line	R (%)	X (%)	Current Limit I <sub>ij</sub> <sup>max</sup>	line	R (%)	X (%)	Current Limit I <sub>ij</sub> <sup>max</sup>
1→2 (PCC)	0.02	0.05	10	9→10	1.41	4.87	4
				10→11	1.15	3.96	2
2→8	0.12	0.42	5	11→12	0.1	0.34	4
2→18	0.19	0.65	5	12→13	1.31	4.52	2
3→6	1.60	5.52	3	12→15	0.13	0.48	2
3→8	0.16	0.56	5	13→14	0.41	1.42	4
3→11	0.21	0.73	5	14→15	1.81	6.23	2
4→5	1.53	5.27	2	15→16	0.44	1.53	2
5→6	0.61	2.12	5	16→17	0.76	2.61	4
6→7	1.25	4.31	3	18→19	1.37	4.72	4
7→15	0.15	0.53	4	18→21	0.13	0.46	5
7→21	0.31	1.06	5	19→20	1.66	5.72	2
8→9	1.06	3.67	2	20→21	0.59	2.02	5

\*All quantities are given in per-unit (p.u.).

\*\*Specified powers are sampled at a specific time:  $t=36$  hr.

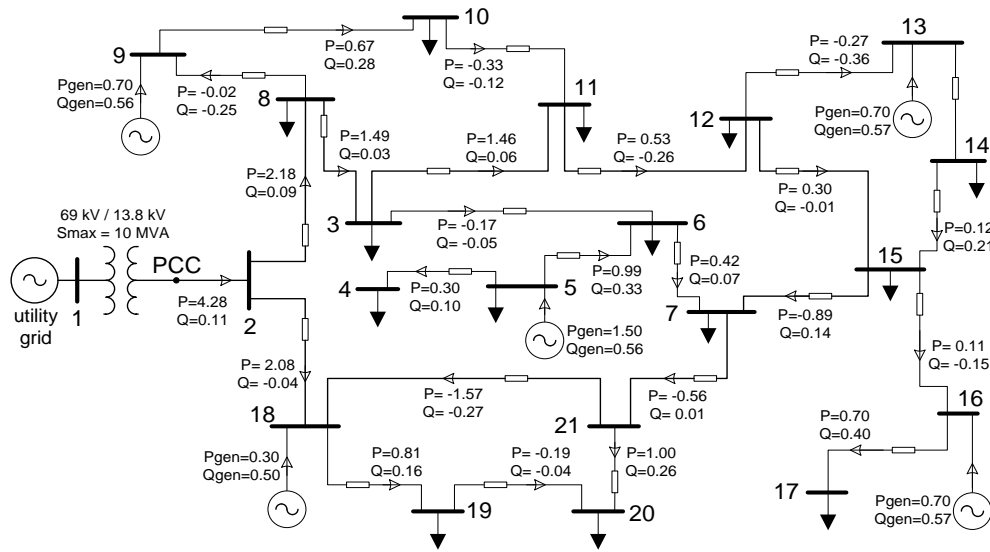


**Fig. 5.** The dynamic programming recursion (illustration). Blue dots mark the Numeric grid. Black 'X' mark possible valid solutions. (a) Backward recursion, computing optimal value for a specific  $Q_{grp1}=1.2$ . (b) Forward recursion. Red circles denote the optimal reactive power at each group.



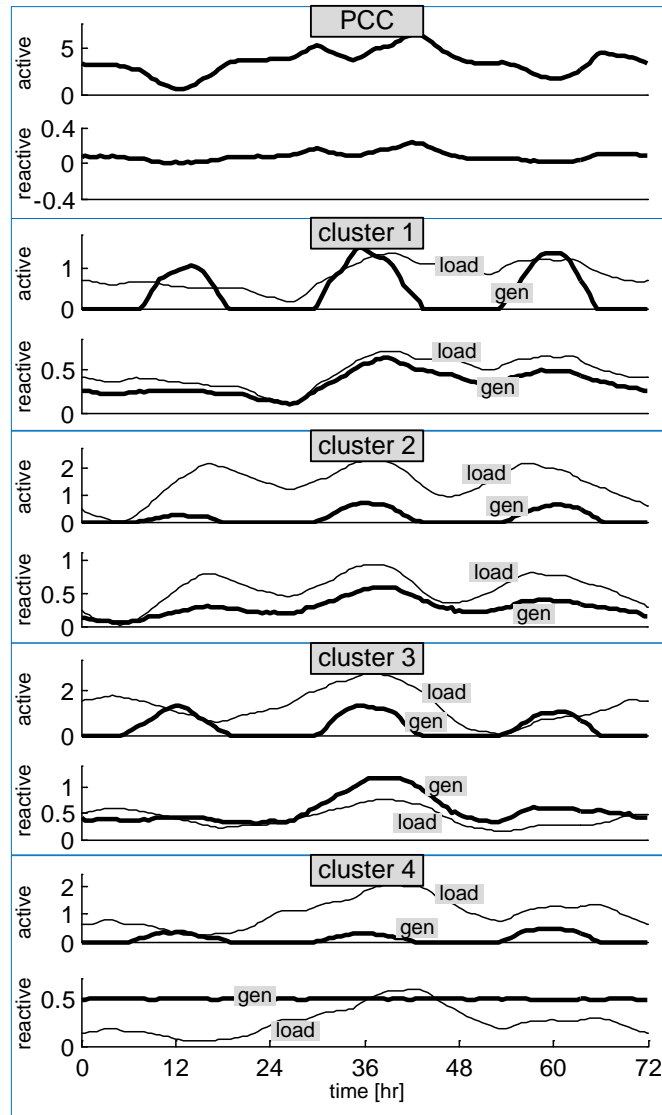
**Fig. 6.** Optimal distribution of reactive power among clusters. Time:  $t=36$  hr. Values are in per-unit. (a) Approximated dynamic programming solution. (b) Exact full-network power flow solution.

Fig. 6 show a comparison of the solution of the proposed test case in an exact calculation and the approximate one according to the proposed dynamic programming procedure as described above. The results of the exact solution are presented in Fig. 6b by using standard Gauss-Seidel algorithm for the entire network. The approximated dynamic programming solution (using the clustering system) is presented in Fig. 6a. Both solutions are calculated for  $t=36$ h. The resulting reactive power outputs of the five generators are tested by a full-network power-flow computation, shown in Fig. 7b, and Fig. 8. The full time-domain solution is shown in Fig. 9. This is created by re-computing the optimal solution at every time point, using current data of loads and power generation.



**Fig. 7** Optimized distribution of reactive power (exact solution, full power flow). Time:  $t=36$  hr. Reactive power is balanced and is nearly zeroed at the PCC.





**Fig. 8** Optimized distribution of reactive power in time domain. Active and reactive powers are shown per cluster. Thin plots mark power consumption (loads). Bold plots mark power generation. Values are in per-unit.

## 6. Additional Microgrid case-study

A second case-study is based on the network in [22]. The network is shown in fig. 9, alongside the optimal reactive power distribution. The microgrid in this case study is a radial network. It is connected to a central PCC (bus 1) through a main transformer of 69/13.8 kV and 15 MVA. The network data and operating point are detailed in Table 2. Clusters in this network are chosen as follows: cluster 1 includes buses 3,7,8,9, cluster 2 includes buses 4,10, cluster 3 includes buses 5,11,12,13, and cluster 4 includes buses 6,14,15. Within each cluster the local power flow is calculated based on the Gauss-Seidel method. Reactive power steps are chosen as  $\Delta Q=0.02$  [p.u.]. The reactive power is optimally distributed among the three generators on buses 9, 13, 15 (generators 2-4) such that a maximal power factor is achieved at

the PCC, considering the constraints of the network and generators as specified in Table 2. The resulting optimal power factor at the PCC is 2.54°.

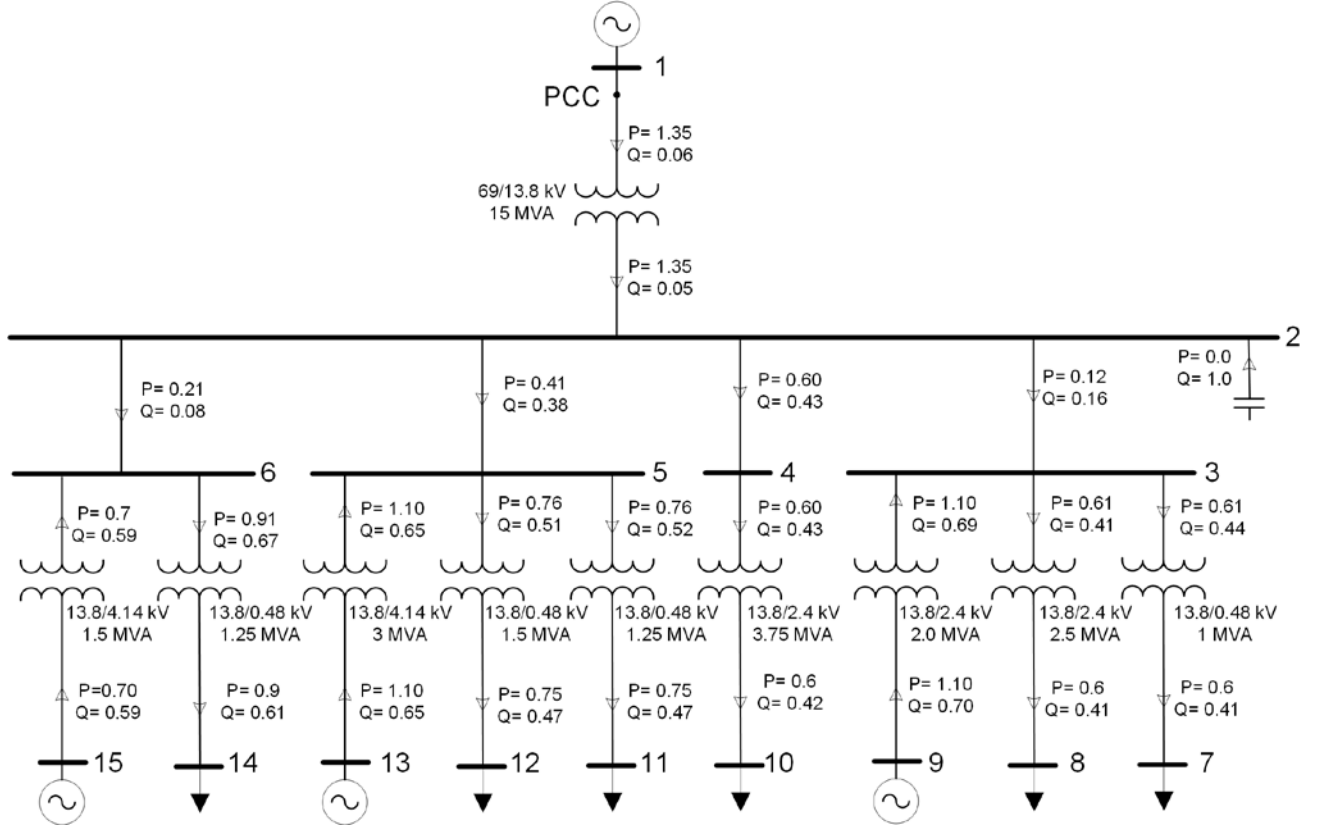


Fig. 9. Optimal Reactive Power Distribution for the Micro-grid case study in [22]

## 7. Comparison to Other Methods

The proposed algorithm has been compared to a standard optimal power flow solver implemented in the MatPower software package [25]. The package provides solver for AC or DC optimal power flow problems, and is based on the primal-dual interior point method, or the trust region based augmented Lagrangian method. We used the AC optimal power flow solver. Using MatPower formulation, the original reactive power optimization problem is formulated as

$$\min_x f(x) \quad (17)$$

Subjected to

$$\begin{aligned} g(x) &= 0 \\ h(x) &\leq 0 \\ x_{min} &\leq x \leq x_{max} \end{aligned} \quad (18)$$

Where  $x$  is given by

$$x = \begin{pmatrix} \angle V_i \\ V_i \\ P_g \\ Q_g \end{pmatrix} \quad (19)$$

In which  $V_i$  are the bus voltages, and  $P_g, Q_g$  are the injected active and reactive power of each generator. The objective function is selected to penalize reactive power consumption at the PCC, and is given by

$$f(x) = c_2 Q_1^2 + c_1 Q_1 + c_3 P_1^2 + c_4 P_1 \quad (20)$$

Where  $c_1, c_2, c_3, c_4$  are positive constants. The original problem is represented by  $c_3=0, c_4=0$ , however these constants may be selected strictly positive to penalize active power consumption at the PCC in addition to reactive power. In addition, the equality constraints  $g(x)=0$  represent the power flow equations and also require zero phase at the PCC ( $\angle V_1=0$ ). The inequality constraints  $h(x) \leq 0$  represent the line flow limits  $|I_{i,j}| \leq I_{i,j}^{max}$ , and the variable limits  $x_{min} \leq x \leq x_{max}$  are

$$\begin{aligned} V_i^{min} &\leq V_i \leq V_i^{max} \\ Q_i^{min} &\leq Q_{g,i} \leq Q_i^{max} \end{aligned} \quad (21)$$

All limit values are provided in Table 1 and Table 2.

The optimization method proposed in this paper is based on dynamic programming. It therefore scans the entire solution space and is guaranteed to converge to the global optimum. In comparison, the Matpower solver is an iterative solver which convergence is not guaranteed. To test the probability of convergence, we tested both solvers with a batch of 1000 random operating points, based on the second microgrid test-case. The variables which are randomized are the active power of each load and generator. Each of them is randomized in the range  $[0.7x \dots 1.3x]$ , where  $x$  is the value provided in Table 2. While the proposed method converges in 100% of the cases, the Matpower solver fails to converge for 153 cases (15.3%). When the algorithm converges, the power factor at the PCC is comparable for both solvers. One advantage of the proposed algorithm is its low computational complexity, which is enabled by the dynamic programming approach. Since the dynamic programming algorithm optimizes the non-convex objective function by parts, instead of scanning the entire solution space, its computational complexity tends to grow linearly with the number of sub problems, instead of exponentially. The low computational complexity is also validated in simulation. It was found that depending on the initial operating point of the system, the average run time was 4.2 ms, as opposed to 89 ms for the cases that converged in Matpower. Both algorithms were tested on the same compute equipped with an i7 Intel processor.

Table 2 - MICROGRID DATA (for the case-study shown in fig. 9)

S <sub>base</sub> = 10 MVA							
Bus Data							
bus	Base voltage kV	Active power demand MW	Reactive power Demand MW	Shunt Susceptance (MVar injected)	Voltage magnitude p.u.	Voltage phase deg.	
1	69	0	0	0	1.0000	0	
2	13.8	0	0	1	0.9986	-0.4338	
3	13.8	0	0	0	0.9973	-0.4332	
4	13.8	0	0	0	0.9936	-0.3884	
5	13.8	0	0	0	0.9922	-0.5444	
6	13.8	0	0	0	0.9976	-0.4501	
7	0.48	0.605	0.405	0	0.9675	-2.3042	
8	2.4	0.605	0.405	0	0.9939	-0.4228	
9	2.4	0	0	0	1.0072	0.1688	
10	2.4	0.6	0.42	0	0.9857	-0.8480	
11	0.48	0.75	0.475	0	0.9614	-2.4891	
12	0.48	0.75	0.475	0	0.9677	-2.0699	
13	0.48	0	0	0	0.9923	-0.5380	
14	0.48	0.9	0.61	0	0.9584	-2.7590	
15	4.14	0	0	0	0.9977	-0.4460	
GENERATORS							
bus	Real power generation MW	Optimal reactive power generation MVar	reactive power range Q <sub>min</sub> ... Q <sub>max</sub>	apparent power limit S <sub>max</sub>	Vol. min. V <sub>min</sub>	Vol. Max V <sub>max</sub>	
1 (PCC)	1.3522	0.0574	-	-	0.95	1.05	
9	1.1	0.704	-1.0 ... 1.0	2	0.95	1.05	
13	1.1	0.648	-0.8 ... 0.8	2	0.95	1.05	
15	0.7	0.595	-1.3 ... 1.3	2	0.95	1.05	
LINES							
line	R % p.u.	X % p.u.	Current Limit I <sub>ij</sub> <sup>max</sup> p.u.	line	R % p.u.	X % p.u.	Current Limit I <sub>ij</sub> <sup>max</sup> p.u.
1→2	0.8180	5.6260	1.5	2→5	6.0650	10.1500	0.5
2→3	3.9760	5.1270	0.5	5→11	8.1600	48.3320	0.125
3→7	8.3140	57.6350	0.1	5→12	6.9030	38.4540	0.15
3→8	4.0220	2.3950	0.25	5→13	0	0.1000	0.3
3→9	2.1000	10.9400	0.2	2→6	3.5640	2.6610	0.5
2→4	6.1410	3.0660	0.5	6→14	8.1600	48.3320	0.125
4→10	2.4400	14.8000	0.375	6→15	0	0.1000	0.15

## 8. Conclusions

This work addresses the optimal distribution of reactive power in microgrids. In contrast to previous gradient based solvers, the proposed approach uses dynamic programming to efficiently scan the entire solution space. As a result, a procedure called MRODP was developed for achieving a global optimal solution in real-time.

The objective is to optimize the power factor at the PCC, while taking into account the voltage current and power constraints of the generators and power lines. A key assumption is that the microgrid is not arbitrarily meshed, but consists of clusters that mutually connect to the PCC. At every recursive step, minimization is carried out over a single variable, so every combination of the reactive controllers is scanned only once. At the end of this process, the partial solutions at every cluster are merged by the recursive Bellman equation to recover the optimal distribution of reactive power, and the optimal reactive power import at the PCC.

Another advantage of the proposed optimization process is that it distributed reactive power as locally as possible among the clusters, and thus minimizing power losses over the internal power lines. In general, reactive power generation and load tends to equalize within the clusters whenever possible. This is demonstrated in Fig. 6: while clusters 1 and 4 the reactive load is balanced by internal generation, cluster 2 lacks reactive power, and is compensated by the excess reactive generation of cluster 3. The locality of distribution is also evident in time domain (Fig. 8): Cluster 1 is always self-supplied, as its reactive generation equalizes its load.

The paper also examines MRODP on two test cases one as in Fig. 7 is a ring type microgrid and the one in Fig. 9 is a radial topology. In both cases the procedure converged for all random chosen operating points.

The optimization method proposed in this paper is based on dynamic programming. It therefore scans the entire solution space and is guaranteed to converge to the global optimum. In comparison, the Matpower solver is an iterative solver which convergence is not guaranteed. To test the probability of convergence, we tested both solvers with a batch of 1000 random operating points, based on the second microgrid test-case. The variables which are randomized are the active power of each load and generator. Each of them is randomized in the range  $[0.7x \dots 1.3x]$ , where  $x$  is the value provided in Table 2. While the proposed method converges in 100% of the cases, the Matpower solver fails to converge for 153 cases (15.3%). When the algorithm converges, the power factor at the PCC is comparable for both solvers. One advantage of the proposed algorithm is its low computational complexity, which is enabled by the dynamic programming approach. Since the dynamic programming algorithm optimizes the non-convex objective function by parts, instead of scanning the entire solution space, its computational complexity tends to grow linearly with the number of sub problems, instead of exponentially. The low computational complexity is also validated in simulation. It was found that depending on the initial operating point of the system, the

average run time was 4.2 ms, as opposed to 89 ms for the cases that converged in Matpower. Both algorithms were tested on the same compute equipped with an i7 Intel processor.

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